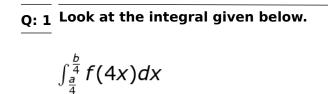
Multiple Choice Questions



If f(x) is continuous for all real values of x, then which of these is equal to the above integral?



Q: 2 What is the value of the following integral?



Q: 3 Varath says the following:

$$\int_{-3}^{3} \left(\sqrt{x^2 - 4} \right) dx = F(3) - F(-3),$$

where F(x) is the antiderivative of $(\sqrt{x^2-4})$.

Which of the following can be said about Varath's statement?

- 1 It is true, as the function is continuous in [-3,3].
- 2 It is true, as per the fundamental theorem of calculus.
- 3 It is false, as the integral is not defined over the interval.
- 4 It is false, as the antiderivative of any function within a square root does not exist in R.

Free Response Questions

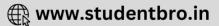
O: 4 Integrate the following function with respect to x.

[1]

$e^{6-\ln x}$

Show your steps.





Q: 5 Given:

$$\int \frac{dx}{f(x)} = \frac{1}{3} \tan^{-1} \left(\frac{x-4}{3} \right) + C$$

where C is an arbitrary constant.

Find f (x). Show your work.

 $\mathbf{Q: 6}$ Check whether the given statement is true or false.

For any function f(x) that satisfies the condition f(-3) = -f(3), $\int_{-3}^{3} f(x) = 0$

Justify your answer.

Q: 7 Ankit's partial solution for a question on integration is given below.

Question: Solve $\int \cos^3 x \sin^2 x \, dx$.

Ankit's solution: Step 1: Let I = $\int \cos^3 x \sin^2 x \, dx$ Step 2: I = $\int \cos^3 x (1 - \cos^2 x) \, dx$ Step 3: I = $\int (\cos^3 x - \cos^5 x) \, dx$ Step 4: Let $\cos x = t$ Step 5: I = $\int (t^3 - t^5) \, dt$

Is Ankit correct? If yes, complete the integration. If no, in which step is the error present? Explain your reasoning.

Q: 8 If h'(x) = g(x) and g is a continuous function for all real values of x, then prove [2] that:

 $\int_{-1}^{1} g(6x) dx = \frac{1}{6}h(6) - \frac{1}{6}h(-6)$

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[1]

[2]

Q: 9 Solve:

$$\int \frac{1}{m^2} \cos^2\left(\frac{1}{m}-1\right) dm$$

Show your steps.

Q: 10 Evaluate the integral:

$$I = \int_0^{\frac{\sqrt{3}}{2}} \left[\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}} dx \right]$$

Show your work.

Q: 11 Solve the following integration and write your answer in its most simplified form. [5]

$$\int \frac{x^2}{e^{2x}} dx$$

Show your steps.

Q: 12 Integrate the given function. Show your steps.

$$\int \cot^{-1}\left(\frac{5}{x}\right) dx$$

$$\frac{\mathbf{Q: 13}}{2e^{x}}$$
 Integrate $\frac{2e^{x}}{(e^{x}-1)(2e^{x}+3)}$.

Show your work.

[3]

[5]

[5]

The table below gives the correct answer for each multiple-choice question in this test.

Q.No	Correct Answers
1	2
2	2
3	3





Q.No	What to look for	Marks
4	Rewrites the given expression as:	0.5
	<u>e⁶</u>	
	Integrates the above expression with respect to x as:	0.5
	$e^{6} \ln x + C$	
	where, C is the constant of integration.	
5	Finds f (x) as: 9 + (x - 4) ²	1
	by differentiating the RHS of the given equation using the differentiation of tan $^{-1}$ x as follows:	
	$\frac{d}{dx}\left(\frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right)\right) = \frac{1}{9+(x-4)^2}$	
6	Writes false.	0.5
	Reasons that this would only be true if $f(-x) = -f(x)$ for every value of x .	0.5
7	Writes that Ankit is incorrect.	0.5
	Writes that Ankit made an error in step 5.	0.5
	Gives a reason. For example, on substituting cos <i>x</i> as <i>t</i> , one also has to substitute <i>dx</i> in terms of <i>dt</i> , which Ankit has not done. Instead, he has simply replaced <i>dx</i> with <i>dt</i> .	1
8	Substitutes $u = 6 x$ and finds du as 6 dx .	0.5
	Finds the limit as:	0.5
	When $x = -1$, then $u = -6$. When $x = 1$, then $u = 6$.	

Q.No	What to look for	Marks
	Rewrites the integral and proves the given statement as:	1
	$\int_{-1}^{1} g(6x) dx$	
	$=\frac{1}{6}\int_{-6}^{6}g(u)du$	
	$= \left[\frac{1}{6}h(u)\right]_{-6}^{6}$	
	$=\frac{1}{6}h(6)-\frac{1}{6}h(-6)$	
9	Substitutes ($\frac{1}{m}$ - 1) as <i>u</i> to get:	0.5
	$du = -\frac{1}{m^2}dm$	
	Rewrites the given integral as:	0.5
	$-\int \cos^2 u du$	
	Substitutes:	0.5
	$\cos^2 u = \frac{(1+\cos 2u)}{2}$	
	in the above integral and rewrites it as follows:	
	$-\frac{1}{2}\int$ (1 + cos 2 <i>u</i>) <i>du</i>	
	Integrates the above integral to get the following expression where C is the arbitrary constant:	1
	$-\frac{1}{2}\left(u+\frac{1}{2}\sin 2u\right)+C$	

Q.No	What to look for	Marks
	Substitutes <i>u</i> as ($\frac{1}{m}$ - 1) in the above expression to get the following expression as the solution:	0.5
	$-\frac{1}{2}\left[\frac{1}{m}-1+\frac{1}{2}\sin 2\left(\frac{1}{m}-1\right)\right]+C$	
10	Takes $u = \sin^{-1} x$	0.5
	Finds du as:	
	$du = \frac{dx}{\sqrt{1 - x^2}}$	
	Finds the change in limit when $x = 0$ and $x = \frac{\sqrt{3}}{2}$ to $u = 0$ and $u = \frac{\pi}{3}$ respectively.	0.5
	Rewrites the given integral using the above substitution and integrates the same as:	1.5
	$I = \int_0^{\frac{\pi}{3}} \frac{u}{\cos^2 u} du$	
	\Rightarrow I = $\int_0^{\frac{\pi}{3}} u \sec^2 u du$	
	\Rightarrow I = u tan u $ _0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}}$ tan u du	
	$\Rightarrow \mathbf{I} = u \tan u \mid_0^{\frac{\pi}{3}} + \log \cos u \mid_0^{\frac{\pi}{3}}$	
	Applies the limit to find the value of the given definite integral as:	0.5
	$\frac{\pi}{\sqrt{3}}$ – log 2	
11	Let $I = \int \frac{x^2}{e^{2x}} dx$	1
	Evaluates integral using formula for integral by parts to get the following:	
	$I = x^2 \left(\int e^{-2x} dx \right) - \int \left[2x \int e^{-2x} dx \right] dx$	

Q.No	What to look for	Marks
	Solves the integration of e^{-2x} by substituting u as (-2 x).	1.5
	Gets <i>dx</i> as $\frac{\cdot 1}{2}$ <i>du</i> .	
	The integration may look as follows:	
	$-\frac{1}{2}\int e^{u}du = -\frac{1}{2}e^{u} + c$ $\Rightarrow \int e^{-2x}dx = -\frac{1}{2}e^{-2x} + c$	
	where c is a constant.	
	Substitutes the value of the above integral in the equation from step 2 to get the following:	0.5
	$I = -\frac{x^2}{2e^{2x}} + \int xe^{-2x}dx$	
	Applies integration by parts to solve the integration of <i>xe</i> ^{-2x} in a similar way as in step 3.	1.5
	The integration may look as follows:	
	Let $h(x) = x$ and $g(x) = e^{-2x}$	
	$\Rightarrow \int x e^{-2x} dx = \frac{x}{-2e^{2x}} - \int \left[\int e^{-2x} dx \right] dx$ $\Rightarrow \int x e^{-2x} dx = -\frac{x}{2e^{2x}} + \frac{1}{2} \int e^{-2x} dx$ $\Rightarrow \int x e^{-2x} dx = -\frac{x}{2e^{2x}} - \frac{1}{4e^{2x}} + c_1$	
	where c_1 is a constant	
	Writes answer as follows:	0.5
	$I = -\frac{2x^2 + 2x + 1}{4e^{2x}} + C$	
	where C is a constant.	

Q.No	What to look for	Marks
12	Uses integration by parts to rewrite the given integral as:	1
	$\cot^{-1}\left(\frac{5}{x}\right)\int dx - \int \left[\frac{d}{dx}\left(\cot^{-1}\left(\frac{5}{x}\right)\right)\int dx\right]dx$	
	Simplifies the differentiation of $\cot^{-1}(\frac{5}{x})$ in the above expression as:	1.5
	$\frac{d}{dx}\cot^{-1}\left(\frac{5}{x}\right)$	
	$= -\frac{1}{1 + \left(\frac{5}{x}\right)^2} \frac{d}{dx} \left(\frac{5}{x}\right)$	
	$=\frac{\frac{5}{x^2}}{1+\left(\frac{5}{x}\right)^2}$	
	$=\frac{5}{x^2+25}$	
	Substitutes the above expression in step 1 and integrates the integral in step 1 to get the following expression:	1
	$x \cot^{-1}\left(\frac{5}{x}\right) - 5 \int \frac{x}{x^2 + 25} dx$	
	Completes integrating the above expression to get:	1.5
	$x \cot^{-1}\left(\frac{5}{x}\right) - \frac{5}{2}\log\left x^2 + 25\right + C$	
	where C is an arbitrary constant.	
	(Award full marks even if modulus is not used in log function as (x^2 + 25) is always positive.)	
13	Considers $e^x = t$ and finds $dx = \frac{dt}{t}$.	1

Q.No	What to look for	Marks
	Writes the function to be integrated as $\int \frac{2}{(t-1)(2t+3)} dt$.	1
	Uses partial fractions to rewrite the function as follows:	1.5
	$\frac{2}{5}\int\left(\frac{1}{t-1}-\frac{2}{2t+3}\right)dt$	
	Integrates the function to obtain the following:	1
	$\frac{2}{5} \left[\ln t - 1 - \ln 2t + 3 \right] + C$	
	where C is the constant of integration.	
	Replaces t with e^x to obtain $\frac{2}{5} \ln \left \frac{e^x - 1}{2e^x + 3} \right + C$.	0.5



